Mangrove's Incentive program

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1 Introduction

1.1 Structure

MangroveDAO introduces MS1 (Mangrove Season 1), a dynamic, point-based leaderboard incentive program designed to encourage active contributions in trading volume and liquidity on the Mangrove DEX. The program is managed by the MangroveDAO.

1.2 Goal

The aim of the incentives program is to bootstrap Mangrove markets by attracting 1) liquidity providers in order to build depth on the Mangrove order books, and 2) liquidity takers to consume the liquidity offered. Takers receive points in proportion to the liquidity they consume. Liquidity providers receive points both for displaying liquidity in the books and having their liquidity consumed. Displaying liquidity is important as for liquidity to be consumed it has to be visible first, either directly on an order book, or indirectly on the charts of a price aggregator (such as Paraswap and 1inch).

1.3 Outline of the document

The next Section (§2) describes the point allocation scheme. Starting values for the various parameters of the scheme are given in §3. These values may change over time. Finally, §4 describes our method to aggregate points allocated to different roles on different markets, and obtain a unified notion of score. The appendix contains a proof of sybil-resistance for the scheme.

2 Structure of the point allocation scheme

Points are allocated to single addresses on a day-to-day basis.

An address can both consume and provide liquidity, and thus receive points for both types of activity.

Point allocations are computed on a per market and per role (taker or maker) basis and then aggregated in a unified score (see §4).

All volumes, whether displayed or consumed, are evaluated in USD, with an exchange rate evaluated at the time/block of the transaction (using a price source to be specified).

Caveat: Volumes consumed by wash-trading, defined as trades between two addresses controlled by the same participant, are *excluded*.

2.1 Takers' Points

For each market and day, takers receive a number of points which is proportional to the total non-excluded volume they consume on that market during that day.

To be eligible, however, a taker has to have consumed at least `minVolumeTaken` during the epoch. The constant `minVolumeTaken` may depend on the market and may change in the future. Starting values are given in §3.

2.2 Makers' Points

Two types of makers are identified, competitive and non-competitive. Competitive makers will be continuously present at the top of the order book keeping a tight spread while giving enough market depth on both sides. Non-competitive makers, on the other hand, may be far from the mid-price, have intermittent presence on the book, and display assymetric liquidity when they are present. The incentives program rewards both types.

2.2.1 Competitive makers

The allocation of points to makers broadly follows dYdX's scheme [1].

The score of a competitive maker m on a market is a product of three factors:

$$s(m) \quad = \quad V(m)\,U(m)\,D(m) \tag{1}$$

The first factor is the *volume factor* $V(m)$. It takes into account the amount of liquidity actually exchanged by m (as opposed to the larger amount displayed by m).

The second factor is the *uptime factor* $U(m)$. It rewards the continuous display of liquidity on the market. The more m is present on the market, the higher its score.

The last factor is the *depth factor* $D(m)$. It also rewards m for displaying liquidity. The larger the amount of liquidity offered, and the closer that liquidity is to the mid-price, the higher the score.

The formulas for each factors is given below. Detailed explanations follow.

The volume factor:

$$V(m) \quad = \quad (\textit{non-excluded volume generated by } m)^v$$

The depth factor is defined as follows:

$$\begin{array}{lcl} D(m,t) & = & \min((\sum_i a_i^t/s_i^t)^d, (\sum_i b_j^t/s_j^t)^d) \\ D(m) & = & \sum_{t\in T} D(m,t) \end{array}$$

where:
- T is a set of random times within the day of interest where the market book is observed
- a_i^t (b_j^t) is the list of asks (bids) placed by m at time t with spread s_i^t
- d, v, u are specific exponents calibrating the various factors

The depth factor inspects the liquidity displayed by m at each snapshot t in T. Not all offers seen on the book are taken into account, though. Their spread has to be small enough and their volume large enough. That is to say we require that:

$$\begin{array}{l} s_i^t \leq \texttt{maxSpread} \\ a_i^t, b_j^t > \texttt{minVolumeDisplayed} \end{array}$$

Because of the 'min' function in the depth factor, displayed liquidity is better rewarded if the liquidity is displayed symmetrically on both sides of the market.

The spread of an offer at price p is defined as $s := p/p_t - 1$ where p_t is a reference price. It is dimension-less.

Spreads smaller than `minSpread` are regularised to `minSpread`.

We assume $d + v = 1$.

The uptime factor derives from the fraction of the time m has an offer eligible for the depth factor:

$$U(m) \quad = \quad (\sum_{t \in T} \mathbf{1}_{\{D(m,t)>0\}})^u$$

The idea is to encourage MMs to provide liquidity homogeneously in time, and, thus u is typically much larger than 1.

As volumes are evaluated in USD, and because $d+v = 1$, the product of the factors $V(m)\, D(m)$ has dimension USD/day.

It is important that the point allocation rewards makers both displaying liquidity and for having that liquidity actually consumed in trades. The choice of the exponents v, d expresses the relative importance of either types of liquidity. As a market matures and its total turnover per day increases [1], the d factor will decrease, and with it the importance of the liquidity displayed.

The choice of $d + v = 1$ is also related to sybil-resistance. See Appendix.

2.2.2 Non competitive makers

Incentivization of all market makers is crucial, irrespective of their order's proximity to the mid-price. Hence introducing non-competitive makers incentivization has an impact not just because of the maximum spread that limits the amount of incentivized users, but also due to the trading volume component. Recall that orders placed significantly away from the spread tend to have a lower likelihood of being executed and thus obtain a zero maker score according to the formula given above in §2.2.1.

While it seems important to reward makers who contribute to less competitive liquidity, we must avoid creating incentives for placing offers too distant from the spread and disadvantaging competitive makers.

Our solution is to assign two scores to makers: one score for competitive makers using the formula in §2.2.1 and another score for non-competitive makers using a variant of this scoring formula. In this variant we drop the maximum spread, and discard the traded volume (equivalently we set its exponent to zero). We also modify the passive liquidity factor with a $\frac{v}{s^3}$ scoring which decreases fast as a function of the spread. This last adjustment prevents attacks where a large amount of liquidity is posted far from the spread and captures all non-competitive makers points.

Let us write:

- $s_{cl}(m)$ and $s_{ncl}(m)$ respectively the competitive and non-competitive liquidity scores computed based on the scoring formula for an address m; $s_{cl}(m)$ is computed using the same formula as before. The function $s_{ncl}(m)$ modifies the competitive score formula, removing the limitation on maximum spread while omitting considerations for traded volume and uptime factors. Additionally, it modifies the scoring of passive liquidity to $\frac{v}{s^3}$, and drops the symmetry requirement. Consequently, at a given time t and with N offers posted by m:

$$s_{ncl}(m) = \sum_{i=0}^{N} \frac{v_i}{s_i^3} \tag{2}$$

- S_{cl} and S_{ncl} respectively the total competitive and non-competitive liquidity score.

- aS_{ncl} the total adjusted non-competitive liquidity score.
- $as_{ncl}(m)$ adjusted non-competitive liquidity score for an address m.

The total adjusted non-competitive liquidity score can depend on the competitive score, for example, since we don't want competitive makers to be disadvantaged, we can cap the amount of points allocated to non-competitive makers and consider that :

$$aS_{ncl} = \alpha S_{cl} \text{ where : } 0 \leq \alpha < 1 \tag{3}$$

Hence the adjusted non-competitive liquidity score for an address m can be expressed as:

$$as_{ncl}(m) = \frac{s_{ncl}(m)}{S_{ncl}} aS_{ncl} \tag{4}$$

The adjusted non-competitive liquidity score can also be expressed as a competitive-to-non-competitive maker ratio.

$$as_{ncl}(m) = s_{ncl}(m) C_{ncl}^{cl} \tag{5}$$

Hence the total score per maker is :

$$s_{total}(m) = s_{cl}(m) + \frac{s_{ncl}(m)}{S_{ncl}} \alpha S_{cl} \tag{6}$$

and the total makers' score is :

$$S_{total} = S_{cl}(1 + \alpha) \tag{7}$$

Remarks :

- The non-competitive makers' score $s_{ncl}(m)$ increases with v and decreases with s. Hence makers are always incentivized to increase their volumes and decrease their spread.
- It is Sybil-proof because of the formula linearity (see Appendix).
- The score's dimension is USD.
- It resists the big liquidity attack: if we take $\frac{v}{s^3}$, 10k@1% = 10M@10% = 10B@100% (that much liquidity is impossible to forward).

3 Starting Parameters

In order to compute makers points on a market, especially the uptime and depth factors U and D, several random snapshots of the market's order book are taken each day. The target rate of sampling is once per minute.

Exponents are set at the following initial values namely: $d = 0.4$, $v = 0.6$, and $u = 5$, across all markets.

Other parameters are shown in Table 3. The Table assumes for illustration purposes that a WETH/USDB and USDB/USDC markets are opened at the start.

One base point, written 1bp, is equal to 10^{-4}.

Market	`minSpread`	`maxSpread`	`minVolumeDisplayed`	`minVolumeTaken`
WETH/USDB	0.1 bp	100 bp	\$100	\$100
USDB/USDC	0.01 bp	10 bp	\$100	\$100

Table 1: Starting values for parameters of the points program

4 Aggregating points across roles and markets

Above, we have shown how to incorporate a depth factor into the point allocation formula for makers, so that they receive points even in the presence of modest order flows (typical of a bootstrapping period). This same factor will also dis-incentivise wash trading, and just-in-time liquidity provision.

Now, we show how to streamline the incentive program, by establishing a unified global ranking system for all users, irrespective of their role(s) or market(s) of interest.

Thus we can flexibly adjust the distribution of points to encourage more activity on a given market and/or in a given role.

The idea is to set beforehand a (per market) variable exchange rate C between maker points and taker points, which is then used ex post to unify maker points and taker points.

4.1 Aggregation over roles

Let a sequence of evaluation times t_i be given, together with time dependent maker-to-taker ratios $C_i > 0$.

For instance, a ratio $C_i = 4$ means that currently a maker point is worth four taker points during that specific period.

Let u be a user address.

Let the last completed period be $J = [t_i, t_{i+1}]$, typically a 24h time interval.

Calculations take place (right) after t_{i+1}.

To simplify: 1) we suppose first that there is only one market; 2) we write C for C_i; 2) we assume a non-zero truncated volume traded during the period -corrected for the `minVolumeTaken` constant (see §2).

Write tp(u), mp(u) for the amount of taker and maker points collected by u during J following the rules of §2.

Write tp(u), mp(u) for the (non-zero) amounts of taker and maker points collected by address u during J according to the rules set out in §2.

Define the following:

$$\begin{aligned} A &= C(\sum_u \text{tp}(u))/(\sum_u \text{mp}(u)) && (8) \\ \text{amp}(u) &:= A \cdot \text{mp}(u) && (9) \end{aligned}$$

One might call A the published-to-trading daily conversion rate, and amp(u) the actual number of maker points accrued by u (for the day).

Note that A is an interesting empirical measure in its own right. The published-to-trading conversion rate is only a valid interpretation for $C = 1$. In the limit when $d = 0$, $v = 1$ (See §2), $A = C$ since for $d = 0$, maker points are exactly the volume generated.

By (Eq 8) the ratio between the total amount of (actual) maker points distributed and the total amount of USD traded (aka taker points) is C, as we wanted it to be.

$$\textstyle\sum_u \mathsf{amp}(u) \quad = \quad A(\sum_u \mathsf{mp}(u)) \quad = \quad C(\sum_u \mathsf{tp}(u))$$

The number of points accrued by u is thus $\mathsf{tp}(u) + A \cdot \mathsf{mp}(u)$ for the period J, and the total number points accrued by all users is:

$$N \quad = \quad (1 + C)(\textstyle\sum_u \mathsf{tp}(u))$$

The idea is that C allows the platform at the beginning of each period to decide which side of the market should be most incentivised; the price to pay for that is that we cannot compute A in advance as its value will result in part form the response of users to the new value of C (and many other independent factors of course).

Each beginning of the day a new value of C may be announced, and each end thereof, the daily conversion rate A is computed, and points assigned.

4.2 Aggregation over markets

To handle the multi-market case, we need to be given relative weights w_k, where k ranges over markets.

Then we write the per-market version of equation (Eq 8):

$$A_k(\sum_u \mathsf{mp}(u,k)) \quad = \quad C_k(\sum_u \mathsf{tp}(u,k)) \tag{10}$$

The rescaled total number of points scored on each market is then $w_k N_k$, and the complete cross-role and cross-market allocation to address u is given by:

$$p(u) \quad = \quad \sum_k w_k(\mathsf{amp}(u,k) + \mathsf{tp}(u,k)) \tag{11}$$

$$\quad = \quad \sum_k w_k(A_k \cdot \mathsf{mp}(u,k) + \mathsf{tp}(u,k)) \tag{12}$$

where A_k is given by (Eq 10).

4.3 Example

We have 2 markets with respective weights and maker/taker ratio:

$$\begin{array}{lll} m_1 & w_1 = 40\% & C_1 = 7/2 \\ m_2 & w_2 = 60\% & C_2 = 5/3 \end{array}$$

We have 4 users with gross points accrued during the time interval of interest (after which we compute the net points)

Score seen as a users $\times$ markets matrix of triplets. Triplets are ordered as (tp, mp, rp).

$$\begin{array}{lll} u_1 & (1500, 0, 0) & (0, 600, 0) \\ u_2 & (0, 500, 0) & (0, 0, 200) \\ u_3 & (0, 0, 800) & (3400, 100, 0) \\ u_4 & (2600, 800, 100) & (0, 0, 0) \end{array}$$

We compute the adjustment coefficients to market points per market:

$$\begin{array}{lcl} A_1 \times 1300 & = & C_1 \times 4100 \\ A_2 \times 700 & = & C_2 \times 3400 \\ A_1 & = & 11.038461538461538 \\ A_2 & = & 8.095238095238097 \end{array}$$

We can now derive aggregate points for each user:

$$\begin{array}{lclcl} p(u_1) & = & w_1(1500 + A_1 \times 0) + w_2(0 + A_2 \times 600) & = & 3514.2857142857147 \\ p(u_2) & = & w_1(0 + A_1 \times 500) + w_2(0 + A_2 \times 0) & = & 2207.692307692308 \\ p(u_3) & = & w_1(0 + A_1 \times 0) + w_2(3400 + A_2 \times 100) & = & 2525.7142857142853 \\ p(u_4) & = & w_1(2600 + A_1 \times 800) + w_2(0 + A_2 \times 0) & = & 4572.307692307692 \end{array}$$

References

A Sybils

It would be unfortunate if the scoring rules incentivised makers to split their liquidity provision into different lots and map those to separate adresses.

Consider a simple symmetric strategy $v@s$ with one offer on each side of the book. Suppose $\lambda \leq 1$ (ie we decrease volume displayed, with $\lambda v \geq$ `minVolumeDisplayed`).

$$\begin{array}{lclcl} D(\lambda v@s) & = & (\lambda v/s)^d & = & \lambda^d D(v@s) \\ V(\lambda v@s) & = & (\lambda v)^v & = & \lambda^v V(v@s) \\ s(\lambda v@s) & = & \lambda^{v+d} s(v@s) & & \end{array}$$

Here we assume that volume taken at a given spread is proportional to volume offered, which should be roughly the case for small volumes (competition for incentives set aside).

Hence the score of λ^{-1} copies of $\lambda v@s$ is:

$$\lambda^{-1} s(\lambda v@s) = \lambda^{v+d-1} s(v@s) \quad (13)$$

In other words $s = \lambda^{v+d-1} s'$.

Therefore homogenous merging is advantageous iff $v + d \geq 1$.

We would like to show that this is true for all splits, not just homogenous ones.

Assume symmetry, so it is enough to look at one side of the book. Assume unit uptime.

The total book of our player is the sum of two separate sets of offers.

Set, accordingly, $S_1 = \sum_i v_i/s_i$, $S_2 = \sum_j v_j/s_j$ the respective pre-scores of the two subsets of offers.

The scores s, s' of the split pair and its merged form are respectively:

$$\begin{array}{rcll} s &=& (\sum_i v_i/s_i)^d V_1^v + (\sum_i v_j/s_j)^d V_2^v & \textit{split} \\ s' &=& (\sum_i v_i/s_i + \sum_i v_j/s_j)^d (V_1 + V_2)^v & \textit{fuse} \end{array}$$

Which we can rewrite as:

$$\begin{array}{rclcl} s &=& S_1^d V_1^v + S_2^d V_2^v &=& S_1^d V_1^v (1 + (S_2/S_1)^d (V_2/V_1)^v) \\ s' &=& (S_1 + S_2)^d (V_1 + V_2)^v &=& S_1^d V_1^v (1 + S_2/S_1)^d (1 + V_2/V_1)^v \end{array}$$

Example (50-50 split of a simple offer): $S_1 = \frac{1}{2}(v/s) = S_2$, $V_1 = \frac{1}{2}V = V_2$ (by symmetry), using the formula above we find $s'/s = 2^{v+d-1}$, ie merge wins iff $v + d \geq 1$, as expected.

But actually, this holds in general. To see this, set $x = S_2/S_1$, $y = V_2/V_1$, and

$$s'/s \quad = \quad g(x, y) \quad := \quad \frac{(1 + x)^d (1 + y)^v}{(1 + x^d y^v)}$$

Merging is advantageous iff $g \geq 1$.

We are interested in finding good pairs (d, v), meaning such that $g(x, y) \geq 1$. (There is no situation where splitting is better than not.)

Observe that:

$$g(x, x) \quad = \quad \frac{(1 + x)^{d+v}}{1 + x^{d+v}}$$

which is ≥ 1 iff $d + v \geq 1$. So that is a necessary condition for being a good pair.

Also:

$$\begin{array}{rcl} g(x, 0) &:=& (1 + x)^d \\ g(0, y) &:=& (1 + y)^v \end{array}$$

So for a pair to be good it must be that $v, d \geq 0$.

When $v+d \geq 1$, $g(x, y) \geq 1$ (Fig. 1, Fig. 2). So the condition is sufficient as well. In other words, general merging is advantageous iff $v + d \geq 1$, and $v, d \geq 0$.

The manner of calculation above does not depend on the v/s choice but only on the g function. The sums S_1, S_2 could use any function of (v, s) to value an offer -eg v/s^k, or $v\,\mathrm{exp}(-cs)$ instead.

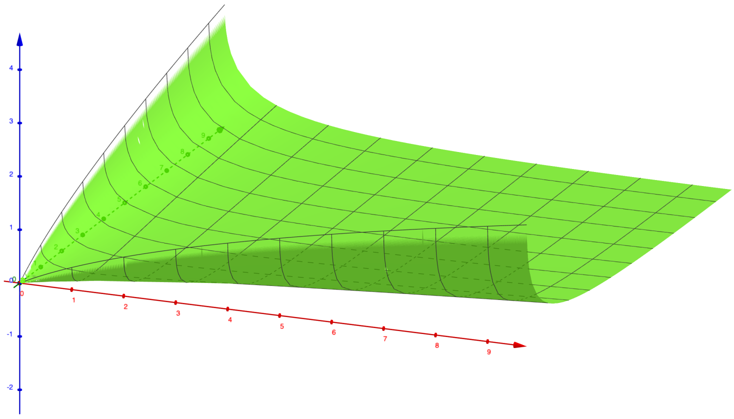

Figure 1: *The difference between the fuse and split scores is always positive for $v + d \geq 1$. See simulation.*

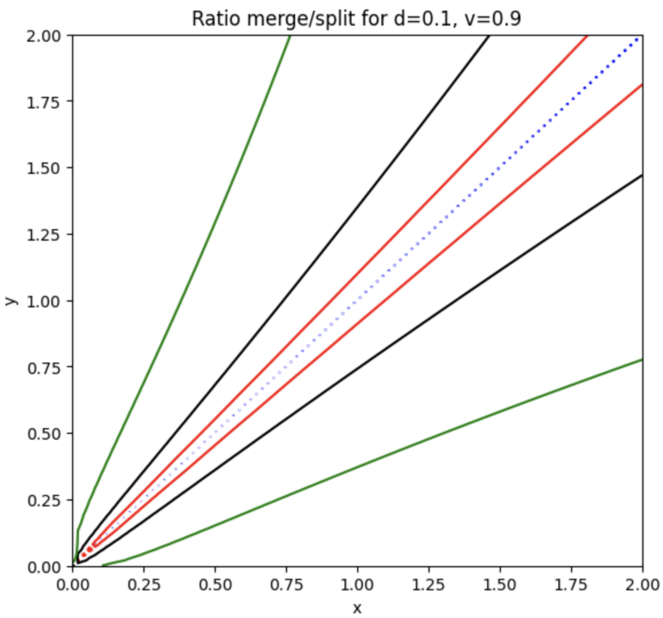

Figure 2: *Contour lines for $g(x, y) - 1$: values = 0 (blue, minimum), 10^{-4} (red), 10^{-3} (black), 10^{-2} (green); $d = 0.1$, $v = 0.9$.*